

ELEVEN DIMENSIONS FROM THE MASSIVE D-2-BRANE

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Abstract

We find an eleven dimensional description of the D-2-brane of the massive type IIA theory as a first step towards an understanding of this theory in eleven dimensions. By means of a world-volume IIA/M theory duality transformation we show that the massive D-2-brane is equivalent to the dimensional reduction of the eleven dimensional membrane coupled to an auxiliary vector field. The role of this vector field is to preserve the invariance under massive gauge transformations in the world-volume and has non-trivial dynamics, governed by a Chern-Simons term proportional to $1/m$.

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1 Introduction

It is well-known that type IIA supergravity can be obtained from eleven dimensional supergravity by a dimensional reduction. The dilaton emerges as a Kaluza-Klein field, related to the compactification radius of the eleventh coordinate as $R_{11} = e^{2\phi/3}$ and, therefore, in the strong coupling limit the theory is eleven dimensional. Hence M-theory (compactified on a circle) has been conjectured to be the strong coupling limit of the type IIA superstring [1, 2]. The p-branes of type IIA supergravity have an alternative interpretation in M-theory² [3, 1, 4], which for the D-branes could be used to deduce their D=10 Lorentz covariant world-volume action. In [5, 6, 7] it was shown that in order to obtain the complete equivalence between the D-2 and D-4-branes of type IIA and the membrane [8] and 5-brane [9] [10, 7] of M-theory a non-trivial transformation involving the abelian world-volume gauge field of the D-brane had to be made. This world-volume transformation was identified as the underlying mapping responsible for IIA/M theory duality [11].

The eleven dimensional interpretation of massive IIA supergravity (an extension of type IIA supergravity including a cosmological constant term) [12, 13] is, however, an open problem. The mass parameter (proportional to the square root of the cosmological constant) can be considered as a RR field of the type IIA theory, since it is the dual of the corresponding RR 9-form³. On the one hand if M-theory is supposed to unify the different string theories we should be able to find an eleven dimensional interpretation of the massive case. However on the other hand, there are several arguments showing that an extension of eleven dimensional supergravity to include a cosmological constant term is not possible [15]. If such an extension exists it must be of some unconventional form.

Our approach to study this problem will be to consider the D-branes of the massive type IIA theory. D-brane actions with $m \neq 0$ have been constructed by T-duality from the type IIB theory [16, 17]. The kappa symmetric extensions have been given in [18].

Our main result will be the derivation of an eleven dimensional description of the massive D-2-brane by performing a duality transformation on its world-volume, in analogous way as an eleven dimensional interpretation could be given to the D-2-brane with $m = 0$ [5, 6]. Now there is an explicit dependence of the world-volume action on the Born-Infeld field that will require a modification of the duality process. The result is that the dual of the vector field will not be a scalar but another vector field. We will show that, still, we can give an eleven dimensional interpretation in the form of an eleven dimensional membrane compactified on a circle, coupled to an auxiliary world-volume vector field whose role is to introduce invariance under massive gauge transformations. This auxiliary field has non trivial dynamics, governed by a Chern-Simons term proportional to $1/m$.

²This applies to all branes but the D-8-brane, which exists for massive type IIA [13].

³The field equations of the RR 9-form fix its dual field strength to a constant with dimensions of mass [14, 13].

2 The massive D-2-brane

The effective action of the massive D-membrane is given by [16, 17, 18]⁴:

$$S = \int d^3x [e^{-\phi} \sqrt{-\det(G_{mn} + B_{mn} + F_{mn})} + \frac{1}{2} \epsilon^{mnp} (\frac{1}{3} C_{mnp} + C_m (F_{np} + B_{np}) + \frac{m}{2} A_m \partial_n A_p)]. \quad (2. 1)$$

(G, B) , $(C_{(1)}, C_{(3)})$ are the NS-NS, RR fields induced in the D-brane from ten dimensions, $F = dA$ is the field strength associated to the BI 1-form and m is the mass parameter corresponding to the dual field strength of the RR 9-form of type IIA. This action is invariant under the RR transformations:

$$\begin{aligned} \delta_{RR} C_m &= \partial_m \alpha \\ \delta_{RR} C_{mnp} &= -\partial_{[m} \alpha B_{np]} \end{aligned} \quad (2. 2)$$

plus the NS-NS transformations:

$$\begin{aligned} \delta C_m &= -m \lambda_m \\ \delta B_{mn} &= -2(\partial_m \lambda_n - \partial_n \lambda_m) \\ \delta C_{mnp} &= m \lambda_{[m} B_{np]} \\ \delta A_m &= 2 \lambda_m. \end{aligned} \quad (2. 3)$$

Notice that when $m = 0$ the RR fields are invariant under the NS-NS transformations, whereas they transform non-trivially for $m \neq 0$. The Chern-Simons term $\frac{m}{4} \epsilon^{mnp} A_m \partial_n A_p$ has to be added to the action to have invariance under (2. 3). This form of the WZ part was shown to be required by T-duality from the type IIB theory [16, 17] and also by kappa symmetry [18]. It can be obtained from the massless D-2-brane by redefining:

$$\begin{aligned} C_m &\rightarrow C_m + \frac{m}{2} A_m \\ C_{mnp} &\rightarrow C_{mnp} - \frac{m}{2} A_{[m} B_{np]} - \frac{3}{2} m A_m \partial_n A_p, \end{aligned} \quad (2. 4)$$

in such a way that the invariance under (2. 3) is introduced⁵. Quantum effects show that the mass parameter and therefore, the cosmological constant, has to be quantized [13, 17]. In [13] this condition is obtained by imposing consistency between T-duality and $SL(2, \mathbb{Z})$ duality of type IIB, and in [17] by demanding independence of the D-brane tension on the compactification radius.

In this paper we will focus on the bosonic part of the action. Kappa symmetric actions have been constructed in [18].

When the mass is equal to zero (2. 1) is invariant under constant translations of the abelian gauge field. One can then construct the so-called first order action, in which dA

⁴We consider a Minkowskian signature space-time.

⁵The invariant NS-NS 2-form is $B_{mn} + \partial_m A_n - \partial_n A_m$.

is replaced by a fundamental two-form whose field strength is imposed to be zero by the introduction of a Lagrange multiplier (a scalar $\tilde{\Lambda}$ in a three-dimensional world-volume). Integrating out the Lagrange multiplier the original action is obtained, whereas the integration over the two-form yields the dual action⁶ [5, 6]:

$$\tilde{S} = \int d^3x [\sqrt{-\det(e^{-2\phi/3}G_{mn} + e^{4\phi/3}(\partial_m\tilde{\Lambda} - C_m)(\partial_n\tilde{\Lambda} - C_n))} + \frac{1}{6}\epsilon^{mnp}(C_{mnp} + 3\partial_m\tilde{\Lambda}B_{np})]. \quad (2.5)$$

This is the Nambu-Goto action of the dimensionally reduced eleven dimensional supermembrane [8], with:

$$\begin{aligned} G_{mn}^{(11)} &= e^{-2\phi/3}G_{mn}^{(10)} + e^{4\phi/3}(\partial_m\tilde{\Lambda} - C_m)(\partial_n\tilde{\Lambda} - C_n) \\ B_{mnp}^{(11)} &= C_{mnp}^{(10)} + 3\partial_m\tilde{\Lambda}B_{np}. \end{aligned} \quad (2.6)$$

It is invariant under the RR gauge transformations (2. 2) if $\tilde{\Lambda}$ transforms as $\delta_{RR}\tilde{\Lambda} = \alpha$, and, trivially, under NS-NS transformations (up to a total derivative).

The world-volume duality transformation provides a way of finding the eleven dimensional (strong coupling) description of the original action. We can then try to follow a similar argument to give an eleven dimensional interpretation to the massive IIA theory. When $m \neq 0$ the explicit dependence of the action (2. 1) on the world-volume gauge field through the Chern-Simons term breaks the symmetry under $A \rightarrow A + \epsilon$ needed to obtain the dual theory in the above procedure. However it is still possible to construct a dual action by building an intermediate Lagrangian from which both the initial and dual theories can be derived.

Let us now motivate the form of this intermediate Lagrangian. The action (2. 5) is not invariant under massive NS-NS transformations. However we can easily modify it in such a way that the invariance is manifest. We just need to replace $\partial_m\tilde{\Lambda}$ by $\partial_m\tilde{\Lambda} + \tilde{A}_m$ with \tilde{A}_m transforming as $\delta\tilde{A}_m = -m\lambda_m$, and add a Chern-Simons term:

$$- \frac{1}{m}\epsilon^{mnp}\tilde{A}_m\partial_n\tilde{A}_p \quad (2.7)$$

to the WZ part. Then the action:

$$\begin{aligned} \tilde{S} &= \int d^3x [\sqrt{-\det(e^{-2\phi/3}G_{mn} + e^{4\phi/3}(\partial_m\tilde{\Lambda} + \tilde{A}_m - C_m)(\partial_n\tilde{\Lambda} + \tilde{A}_n - C_n))} + \\ &\quad \frac{1}{6}\epsilon^{mnp}(C_{mnp} + 3(\partial_m\tilde{\Lambda} + \tilde{A}_m)B_{np}) - \frac{1}{m}\epsilon^{mnp}\tilde{A}_m\partial_n\tilde{A}_p] \end{aligned} \quad (2.8)$$

is invariant under the two transformations (2. 2) and (2. 3) plus $\delta_{RR}\tilde{\Lambda} = \alpha$ and $\delta\tilde{A}_m = -m\lambda_m$. Notice that \tilde{A}_m can be taken as a coexact 1-form, since the NS-NS transformations (2. 3) are non-trivial only if λ is coexact. If λ is exact⁷ the NS-NS backgrounds are invariant⁸ and the RR fields transform as (2. 2) with α depending on the mass, and this transformation

⁶An alternative way of deriving dual actions using an auxiliary metric has been derived in [19].

⁷We consider topologically trivial world-volumes.

⁸ \tilde{A}_m transforms with a total derivative but it doesn't contribute to the action.

is already cancelled by $\delta_{RR}\tilde{\Lambda} = \alpha$. We can then work with a fundamental 1-form $\tilde{V}_m \equiv \partial_m\tilde{\Lambda} + \tilde{A}_m$ whose exact component is given by the differential of the eleventh coordinate and the coexact one by the auxiliary field \tilde{A}_m .

We now show that in fact (2. 8) is equivalent to the massive D-2-brane action under a world-volume duality transformation. We just need to realize that it can be obtained from the intermediate action:

$$S_I = \int d^3x [\sqrt{-\det(e^{-2\phi/3}G_{mn} + e^{4\phi/3}(\tilde{V}_m - C_m)(\tilde{V}_n - C_n))} + \frac{1}{6}\epsilon^{mnp}C_{mnp} + \frac{1}{2}\epsilon^{mnp}\tilde{V}_m B_{np} + \frac{m}{4}\epsilon^{mnp}A_m\partial_n A_p + \frac{1}{2}\epsilon^{mnp}\tilde{V}_m(\partial_n A_p - \partial_p A_n)] \quad (2. 9)$$

after integration over the auxiliary field A_m . The equation of motion for A_m gives the constraint $\epsilon^{mnp}(\partial_n \tilde{V}_p + \frac{m}{2}\partial_n A_p) = 0$, which substituted in (2. 9) gives the action (2. 8) up to a total derivative.

If instead we integrate out \tilde{V}_m it is easy to check that the action for the massive D-2-brane is obtained. For this purpose it is helpful to write:

$$\sqrt{-\det(e^{-2\phi/3}G_{mn} + e^{4\phi/3}(\tilde{V}_m - C_m)(\tilde{V}_n - C_n))} = \sqrt{-\det g} \sqrt{1 + e^{4\phi/3}(\tilde{V}_m - C_m)(\tilde{V}^m - C^m)} \quad (2. 10)$$

where $g_{mn} = e^{-2\phi/3}G_{mn}$. Therefore when $m \neq 0$ the dual of the vector field A_m is also a vector field⁹ \tilde{V}_m , that can be decomposed into an exact component, playing the role of the (differential of the) eleventh coordinate, plus a coexact component. (2. 8) then provides an eleven dimensional description of the massive D-2-brane as the action of the eleven dimensional membrane compactified on a circle, coupled to an auxiliary world-volume field required to introduce invariance under massive gauge transformations. This field has non-trivial dynamics, dictated by the Chern-Simons term proportional to $1/m$. As a check, when the mass is sent to zero the Chern-Simons term in (2. 8) vanishes, implying that $\tilde{A}_m = 0$ (since it is a coexact 1-form), and the action for the dimensional reduction of the eleven dimensional membrane (2. 5) is recovered.

It is worth noting that (2. 8) can be obtained from the massless dual action (2. 5) after the redefinitions:

$$\begin{aligned} C_m &\rightarrow C_m - \tilde{A}_m \\ B_{mn} &\rightarrow B_{mn} - \frac{2}{m}(\partial_m \tilde{A}_n - \partial_n \tilde{A}_m) \\ C_{mnp} &\rightarrow C_{mnp} + \tilde{A}_{[m} B_{np]} - \frac{6}{m}\tilde{A}_m \partial_n \tilde{A}_p \end{aligned} \quad (2. 11)$$

are made. Comparing these expressions with (2. 4) we see that \tilde{A} plays the same role than the original BI field A (note the different scalings with the mass). In both the original and

⁹This is a well-known result in topologically massive three dimensional gauge theories (see for instance [20]). Our intermediate action (2. 9) is the generalization to the Born-Infeld Lagrangian of the intermediate action presented there.

dual theories these fields need to be introduced in order to have invariance under the NS-NS transformations (2. 3).

We give now some speculations about the possible space-time interpretation of the auxiliary field \tilde{A} . The redefinitions (2. 4) and (2. 11) resemble the transformations of the ten dimensional background fields:

$$\begin{aligned} B' &= B - \frac{2}{m}dC_{(1)} \\ C'_{(3)} &= C_{(3)} + 3C_{(1)}B - \frac{3}{m}C_{(1)}dC_{(1)}, \end{aligned} \tag{2. 12}$$

required to formulate Romans' massive IIA supergravity [12] in a way that the invariance under the gauge symmetries:

$$\begin{aligned} \delta C_{(1)} &= -m\lambda \\ \delta B &= -2d\lambda \\ \delta C_{(3)} &= 3m\lambda B \end{aligned} \tag{2. 13}$$

is manifest. The RR 1-form is then said to play the role of a Stueckelberg field, introducing the symmetry under massive gauge transformations. The NS-NS 2-form acquires mass by absorbing the RR 1-form, in a Higgs-type mechanism (see [12, 13] for details). The space-time transformations (2. 12) are obtained in the world-volume if the induced RR 1-form absorbs the auxiliary vector field A or \tilde{A} (depending on whether we are in the original or dual theories). Perhaps this could give a hint on the possible space-time interpretation of this field.

The obvious next step would be to find the eleven dimensional supergravity extension containing (2. 8) as a solution. An idea for this purpose is to find the kappa symmetric extension of this action, since for $m = 0$ it is known that kappa symmetry of the eleven dimensional membrane implies that the backgrounds must satisfy the field equations of eleven dimensional supergravity [8]. Along these lines it has been shown in [21] that the field equations of massive IIA supergravity are implied by kappa symmetry of the massive D-2-brane. We leave this point for further investigation.

3 Conclusions

We have seen that the massive D-membrane of the type IIA theory can be interpreted as the eleven dimensional membrane compactified on a circle, coupled to an auxiliary world-volume vector field needed to introduce invariance under the massive gauge transformations induced in the world-volume. This field has non-trivial dynamics, governed by a Chern-Simons term proportional to $1/m$. Its physical meaning remains an open problem which we hope to address in a near future.

A similar type of effective action induced from eleven dimensions with an additional vector field has been presented in [22] in the context of the eleven dimensional Kaluza-Klein

monopole. This solution has eight translational isometries but only seven can be interpreted as world-volume directions [23]. This implies that there is an extra scalar in the world-volume action, that needs to be removed in order to have the correct degrees of freedom. The mechanism proposed in [22] is to gauge the isometry under translations on the eighth direction by introducing a vector field in the world-volume action. It could be interesting to analyze if a similar type of interpretation could be applicable to our effective action.

The equivalence between the original and dual actions that we have presented relies on a saddle-point approximation. This was also the case for the D-2-brane/eleven dimensional membrane duality of [5]. In that case it was possible to show the equivalence of the two theories at the level of their partition functions by formulating the duality as a canonical transformation in the phase space associated to the BI field of the D-2-brane [6]. It is yet unclear to us whether this is also the case in the present theories.

Finally, it would be interesting to find similar eleven dimensional descriptions for other D-branes of massive type IIA, and in particular for the D-8-brane. This could shed some light on the determination of the action of the eleven dimensional 9-brane [13, 24, 23].

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